

- $c : E \rightarrow \mathbb{R}_0^+$ : capacity of an edge
- $e : V \rightarrow \mathbb{R}_0^+$ : excess (too much flow in one node)
- $r_f : V \times V \rightarrow \mathbb{R}$ ,  $r_f(u, v) := c(u, v) - f(u, v)$ : remaining capacity
- $dist : V \rightarrow \mathbb{N}$ : the label (imagine this as height)

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**Algorithm 1** Algorithm of Goldberg and Tarjan
 

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function PUSHRELABEL(Network  $N(D, s, t, c)$ )
  for all  $(v, w) \in (V \times V \setminus E)$  do           ▷ If an edge is not in  $D = (V, E)$ ,
     $c(v, w) \leftarrow 0$                              ▷ then its capacity is 0
  end for

  for all  $(v, w) \in V \times V$  do                   ▷ At the beginning, every edge
     $f(v, w) \leftarrow 0$                                ▷ has flow=0
     $r_f(v, w) \leftarrow c(v, w)$                    ▷ flow=max in the residualgraph
  end for

   $dist(s) \leftarrow |V|$ 
  for all  $v \in V \setminus \{s\}$  do
     $f(s, v) \leftarrow c(s, v)$            ▷ Push maximum flow out at the beginning
     $r(v, s) \leftarrow c(v, s) - f(v, s)$ 
     $dist(v) \leftarrow 0$ 
     $e(v) \leftarrow c(s, v)$                  ▷  $v$  has too much flow
  end for

  while  $\exists v \in V : \text{ISACTIVE}(v)$  do
    if  $\text{ISPUSHOK}(v)$  then
      PUSH( $v$ )
    end if
    if  $\text{ISRELABELOK}(v)$  then
      RELABEL( $v$ )
    end if
  end while

  return  $f$                                      ▷ Maximaler Fluss
end function

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function PUSH(Node  $v$ , Node  $w$ )
   $\Delta \leftarrow \min \{ e(v), r_f(v, w) \}$ 
   $f(v, w) \leftarrow f(v, w) + \Delta$ 
   $f(w, v) \leftarrow f(w, v) - \Delta$ 
   $r_f(v, w) \leftarrow r_f(v, w) - \Delta$ 
   $r_f(w, v) \leftarrow r_f(w, v) + \Delta$ 
   $e(v) \leftarrow e(v) - \Delta$ 
   $e(w) \leftarrow e(w) + \Delta$ 
end function

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function RELABEL(Node  $v$ )
  if  $\{ w \in V \mid r_f(v, w) > 0 \} == \emptyset$  then
     $dist(v) \leftarrow \infty$ 
  else
     $dist(v) \leftarrow \min \{ dist(w) + 1 \mid w \in V : r_f(v, w) > 0 \}$ 
  end if
end function

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function ISACTIVE(Node  $v$ )
  return  $(e(v) > 0) \wedge (dist(v) < \infty)$ 
end function

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function ISRELABELOK(Node  $v$ )
  return  $\text{ISACTIVE}(v) \bigwedge_{w \in \{ w \in V \mid r_f(v, w) > 0 \}} (dist(v) \leq dist(w))$ 
end function

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function ISPUSHOK(Node  $v$ )
  return  $\text{ISACTIVE}(v) \wedge (e(v) > 0) \wedge (dist(v) == dist(w) + 1)$ 
end function

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